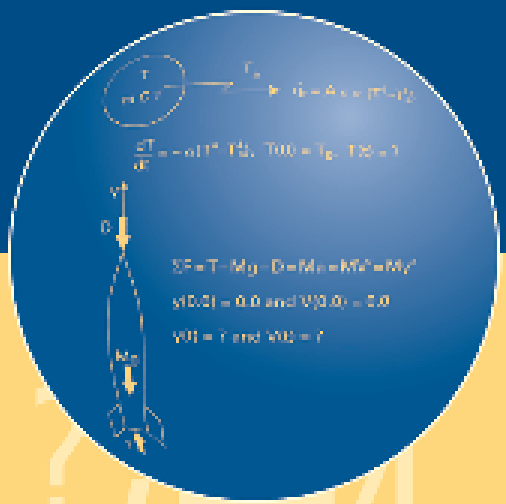
Numerical Methods for Engineers and Scientists

Second Edition
Revised and Expanded



Joe D. Hoffman

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Second Edition Revised and Expanded

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To Cynthia Louise Hoffman

Preface

The second edition of this book contains several major improvements over the first edition. Some of these improvements involve format and presentation philosophy, and some of the changes involve old material which has been deleted and new material which has been added.

Each chapter begins with a chapter table of contents. The first figure carries a sketch of the application used as the example problem in the chapter. Section 1 of each chapter is an introduction to the chapter, which discusses the example application, the general subject matter of the chapter, special features, and solution approaches. The objectives of the chapter are presented, and the organization of the chapter is illustrated pictorially. Each chapter ends with a summary section, which presents a list of recommendations, dos and don'ts, and a list of what *you should be able to do after studying the chapter.* This list is actually an itemization of what the student should have learned from the chapter. It serves as a list of objectives, a study guide, and a review guide for the chapter.

Chapter 0, Introduction, has been added to give a thorough introduction to the book and to present several fundamental concepts of relevance to the entire book.

Chapters 1 to 6, which comprise Part I, Basic Tools of Numerical Analysis, have been expanded to include more approaches for solving problems. Discussions of pitfalls of selected algorithms have been added where appropriate. Part I is suitable for second-semester sophomores or first-semester juniors through beginning graduate students.

Chapters 7 and 8, which comprise Part II, Ordinary Differential Equations, have been rewritten to get to the methods for solving problems more quickly, with less emphasis on theory. A new section presenting extrapolation methods has been added in Chapter 7. All of the material has been rewritten to flow more smoothly with less repetition and less theoretical background. Part II is suitable for juniors through graduate students.

Chapters 9 to 15 of the first edition, which comprised Part III, Partial Differential Equations, has been shortened considerably to only four chapters in the present edition. Chapter 9 introduces elliptic partial differential equations. Chapter 10 introduces parabolic partial differential equations, and Chapter 11 introduces hyperbolic partial differential equations. These three chapters are a major condensation of the material in Part III of the first edition. The material has been revised to flow more smoothly with less emphasis on theoretical background. A new chapter, Chapter 12, The Finite Element Method, has been added to present an introduction to that important method of solving differential equations.

A new section, Programs, has been added to each chapter. This section presents several FORTRAN programs for implementing the algorithms developed in each chapter to solve the example application for that chapter. The application subroutines are written in

a form similar to pseudocode to facilitate the implementation of the algorithms in other programming languages.

More examples and more problems have been added throughout the book.

The overall objective of the second edition is to improve the presentation format and material content of the first edition in a manner that not only maintains but enhances the usefullness and ease of use of the first edition.

Many people have contributed to the writing of this book. All of the people acknowledged in the Preface to the First Edition are again acknowledged, especially my loving wife, Cynthia Louise Hoffman. My many graduate students provided much help and feedback, especially Drs. D. Hofer, R. Harwood, R. Moore, and R. Stwalley. Thanks, guys. All of the figures were prepared by Mr. Mark Bass. Thanks, Mark. Once again, my expert word processing specialist, Ms. Janice Napier, devoted herself unsparingly to this second edition. Thank you, Janice. Finally, I would like to acknowledge my colleague, Mr. B. J. Clark, Executive Acquisitions Editor at Marcel Dekker, Inc., for his encouragement and support during the preparation of both editions of this book.

Joe D. Hoffman

Contents

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Introduction

This Introduction contains a brief description of the objectives, approach, and organization of the book. The philosophy behind the Examples, Programs, and Problems is discussed. Several years' experience with the first edition of the book has identified several simple, but significant, concepts which are relevant throughout the book, but the place to include them is not clear. These concepts, which are presented in this Introduction, include the definitions of significant digits, precision, accuracy, and errors, and a discussion of number representation. A brief description of software packages and libraries is presented. Last, the Taylor series and the Taylor polynomial, which are indispensable in developing and understanding many numerical algorithms, are presented and discussed.

0.1 OBJECTIVE AND APPROACH

The objective of this book is to introduce the engineer and scientist to numerical methods which can be used to solve mathematical problems arising in engineering and science that cannot be solved by exact methods. With the general accessibility of high-speed digital computers, it is now possible to obtain rapid and accurate solutions to many complex problems that face the engineer and scientist.

The approach taken is as follows:

1. Introduce a type of problem.

2. Present sufficient background to understand the problem and possible methods of solution.
3. Develop one or more numerical methods for solving the problem.
4. Illustrate the numerical methods with examples.

In most cases, the numerical methods presented to solve a particular problem proceed from simple methods to complex methods, which in many cases parallels the chronological development of the methods. Some poor methods and some bad methods, as well as good methods, are presented for pedagogical reasons. Why one method does not work is almost as important as why another method does work.

0.2 ORGANIZATION OF THE BOOK

The material in the book is divided into three main parts:

I. Basic Tools of Numerical Analysis
II. Ordinary Differential Equations
III. Partial Differential Equations

Part I considers many of the basic problems that arise in all branches of engineering and science. These problems include: solution of systems of linear algebraic equations, eigenproblems, solution of nonlinear equations, polynomial approximation and interpolation, numerical differentiation and difference formulas, and numerical integration. These topics are important both in their own right and as the foundation for Parts II and III.

Part II is devoted to the numerical solution of ordinary differential equations (ODEs). The general features of ODEs are discussed. The two classes of ODEs (i.e., initial-value ODEs and boundary-value ODEs) are introduced, and the two types of physical problems (i.e., propagation problems and equilibrium problems) are discussed. Numerous numerical methods for solving ODEs are presented.

Part III is devoted to the numerical solution of partial differential equations (PDEs). Some general features of PDEs are discussed. The three classes of PDEs (i.e., elliptic PDEs, parabolic PDEs, and hyperbolic PDEs) are introduced, and the two types of physical problems (i.e., equilibrium problems and propagation problems) are discussed. Several model PDEs are presented. Numerous numerical methods for solving the model PDEs are presented.

The material presented in this book is an introduction to numerical methods. Many practical problems can be solved by the methods presented here. Many other practical problems require other or more advanced numerical methods. Mastery of the material presented in this book will prepare engineers and scientists to solve many of their everyday problems, give them the insight to recognize when other methods are required, and give them the background to study other methods in other books and journals.

0.3 EXAMPLES

All of the numerical methods presented in this book are illustrated by applying them to solve an example problem. Each chapter has one or two example problems, which are solved by all of the methods presented in the chapter. This approach allows the analyst to compare various methods for the same problem, so accuracy, efficiency, robustness, and ease of application of the various methods can be evaluated.