Quantum Fields

N. N. BOGOLIUBOV
D.V. SHIRKOV

N. N. Bogoliubov and D. V. Shirkov, Quantum Fields

First English Edition, 1982
Originally published in 1980 as Квантовые поля
by Izdatel'stov "Nauka," Moscow

Library of Congress Cataloging in Publication Data

Bogoliubov, N. N. (Nikolaĭ Nikolaevich), 1909–
Quantum fields.

Translation of: Kvantovye polia.
Bibliography: p.
Includes index.
1. Quantum field theory. I. Shirkov, D. V. (Dmitriĭ Vasil'evich) II. Title.
QC174.45.B5813 1982 530.1'43 82-4366
ISBN 0-8053-0983-7

Printed in the United States of America

ABCDEFGHIJ-HA-898765432

CONTENTS

PREFACE

The main purpose of this book is to provide graduate students in physics with the necessary minimum of information on the fundamentals of modern quantum field theory.

It may turn out to be sufficient both for theoreticians, specializing in nuclear physics, quantum statistics and other fields, in which quantum field methods are utilized and which are based on quantum concepts, and also for experimental physicists in the fields of nuclear and high-energy physics. For the latter category of readers the present book should be supplemented by a course on particle physics and particle interactions. At the same time the book can be recommended as an introductory text for persons intending to work in the field of quantum field theory and of the theory of elementary interactions.

The material in this book corresponds to a course lasting one academic year. Our personal experience testifies that parallel practical studies at seminars are extremely desirable. For this purpose part of the technical material has been assembled at the end of the book in the form of Appendices. There, also, sets of exercises and problems, gathered together as assignments corresponding to chapters of the main text, are given.

The authors are grateful to the editor of this book D. A. Slavnov, to the reviewers M. A. Brown, L. V. Prokhorov, K. A. Ter-Martirosyan, and also to B. M. Barbashov, B. V. Medvediev, and N. M. Shumeiko for valuable comments on the typescript of the book.

N. N. Bogoliubov
D. V. Shirkov

PREFACE TO THE ENGLISH-LANGUAGE EDITION

This book is a text on the fundamentals of quantum field theory and renormalized perturbation theory (RPT). The traditional field of application of the latter for a long time was limited to quantum electrodynamics. During recent years, due to the creation of a unified theory of electroweak interactions and to the successes of quantum chromodynamics it has become clear that the physical scope of RPT is much wider. However, in the study of the quark–gluon interaction, as well as of possible mechanisms of the grand unification of interactions, a decisive part is played by the simultaneous use of results of RPT and of the apparatus of the renormalization-group method.

Therefore we have written a special Appendix IX, "The renormalization group," for the English-language edition of our book. Besides this, small editorial changes and corrections of noticed misprints have been made.

N. N. Bogoliubov
D. V. Shirkov

Quantum Fields

SOME BASIC CONCEPTS AND NOTATION

We shall take all components of four-vectors to be real. The four-vector $a=(a^0, \boldsymbol{a})$ consisting of the zero component a^0 and the space vector $\boldsymbol{a}$ will, according to tradition, be called a contravariant 4-vector, and we shall denote its components by upper indices

$$a^\nu=(a^0,\ a^1,\ a^2,\ a^3).$$

We define the product of two vectors a and b as

$$ab=a^0b^0-\boldsymbol{ab}=a^0b^0-a^1b^1-a^2b^2-a^3b^3.$$

It is conveniently written in the form

$$ab=\sum_{\mu,\ \nu} g_{\mu\nu}a^\mu b^\nu \qquad (\mu,\ \nu=0,\ 1,\ 2,\ 3),$$

where $g_{\mu\nu}$ is a diagonal metric tensor

$$g_{\mu\nu}=0 \text{ when } \mu\neq\nu,\quad g_{00}=1;\quad g_{11}=g_{22}=g_{33}=-1,$$

differing in sign from the well-known Minkowski tensor. The transition from contravariant components a^ν to covariant ones a_μ (i.e. lowering of indices) is accomplished with the aid of the metric tensor:

$$a_\mu=g_{\mu\nu}a^\nu,\qquad a^\mu=g^{\mu\nu}a_\nu,\qquad g_{\mu\nu}=g^{\mu\nu},$$

that is,

$$a_0=a^0,\qquad a_k=-a^k\quad (k=1,\ 2,\ 3),\qquad a_\nu=(a^0,\ -\boldsymbol{a}).$$

Here and below, summation over twice repeated indices is implied in all cases, and we will omit the summation sign. Indices representing summation over the three space components are denoted by Latin letters, taken from the middle of the alphabet:

$$ab = a_n b_n = a^k b^k = a_1 b_1 + a_2 b_2 + a_3 b_3,$$

while indices representing summation over all four components (0, 1, 2, 3) are denoted by Greek letters:

$$ab = a^\nu b_\nu = a_\mu b^\mu = \sum_{\mu,\nu} g^{\mu\nu} a_\mu b_\nu .$$

Occasionally, in order to simplify the form of a cumbersome expression, we shall lower or raise both Greek indices, that is

$$A_\nu B_\nu = A^\mu B^\mu \equiv AB = A_\nu B^\nu,$$

$$F_{\mu\nu} F_{\mu\nu} \equiv F_{\mu\nu} F^{\mu\nu} = \sum_{\mu,\nu} F_{\mu\nu} F^{\mu\nu}$$

Thus, the presence of two identical Greek indices in different factors *always* implies covariant summation independently of the positions of the indices.

Indices associated with internal symmetry groups (for example, isotopic indices) are denoted, as a rule, by latin letters taken from the beginning of the alphabet (a, b, . . .).

The symbol $\hat{a}$ stands for the contraction of the four-vector components a_ν with the Dirac matrices

$$\hat{a} = a^\nu \gamma_\nu = a_\nu \gamma^\nu .$$

For derivatives with respect to covariant and contravariant components the following abbreviated notation is often used:

$$\frac{\partial u}{\partial x_\nu} = \partial^\nu u = u^{;\nu}, \qquad \frac{\partial \varphi_a}{\partial x^\nu} = \partial_\nu \varphi_a = \varphi_{a;\,\nu} .$$

Naturally, we have

$$\varphi_a^{;\mu} = g^{\mu\nu} \varphi_{a;\,\nu} .$$

We represent the D'Alembert operator

$$\square = \Delta - \partial_0^2$$

in the form

$$\square = - \partial^\nu \partial_\nu .$$

Throughout the book the so-called *rational system of units* is used, in which the velocity of light and Planck's constant are taken equal to unity, i.e., $c = \hbar = 1$. In this system energy and momentum have the dimension of mass, or of a reciprocal length, and the time $x_0 = t$ has the dimension of length

$$[E] = [p] = m = l^{-1}, \qquad [x_0] = [x] = l = m^{-1}.$$

As a rule, the formulae for the four-dimensional Fourier transform are written in the form

$$f(x) \sim \int e^{-ipx} \tilde{f}(p)\, dp, \qquad \tilde{f}(p) \sim \int e^{ipx} f(x)\, dx.$$

The sign of the exponent is chosen to be such that the time component corresponds to the quantum-mechanical expression

$$f(x^0, \mathbf{x}) = f(t, \mathbf{x}) \sim \int e^{-iEt} \tilde{f}(E, \mathbf{x})\, dE.$$

The three-dimensional Fourier transform correspondingly is

$$\varphi(\mathbf{x}) \sim \int e^{i\mathbf{p}\mathbf{x}} \tilde{\varphi}(\mathbf{p})\, d\mathbf{p}, \qquad \tilde{\varphi}(\mathbf{p}) \sim \int e^{-i\mathbf{p}\mathbf{x}} \varphi(\mathbf{x})\, d\mathbf{x}.$$

An exception will be made for the positive-frequency parts of field functions and the positive-frequency parts of the Green's functions. The normalizing factors of the Fourier transforms (powers of 2π) are chosen in different ways in various parts of this book.

Within each section single numbering of formulae is used: (1), (2), (3), . . . , which is used directly for references to formulae within the given section. Double numbering, like (8.12), (AII.6), indicates references to formulae of other sections. The first symbol points to the number of the section or appendix (Section 8, Appendix II), and the second one to the ordinal number of the formula in the section.

Bibiliographical references are denoted by the years printed after the author name. For example, "Medvediev (1977)" denotes a reference to the book by B. V. Medvediev, published in 1977, the full bibliographical title of which is presented in the list of references at the end of the present book. The only exception is made for the third edition of our book *Introduction to the Theory of Quantized Fields*, which will be referred to as the *Introduction.*

1. PARTICLES AND FIELDS

1.1. Particles and their main properties. Quantum field theory is a physical theory of elementary particles and their interactions. It is based on the connection arising between relativistic particles and quantum fields, when classical fields are quantized. The properties of the quantized fields closely correspond to the properties of the particles. Therefore, we shall first of all make a list of the main properties of particles.

An important attribute of a relativistic particle is its rest mass m. The theory of relativity relates the mass m to the particle energy E and its momentum p by the well known relation:

$$E^2 - c^2p^2 = m^2c^4.$$

In the rational system of units this relation assumes the form

$$E^2 - p^2 = m^2.$$

In this system mass can be measured in energy units. For the mass of the electron m_e we have* $m_e = 9.109534(47) \times 10^{-28}$ g $= 0.5110034(14)$ MeV. The proton mass $m_p = 938.2796(27)$ MeV $\simeq 0.94$ GeV, and so on.

The second significant characteristic of a particle is its spin (i.e., its intrinsic angular momentum). In accordance with general theorems of quantum mechanics the spins of particles turn out to be quantized—their absolute values are integer multiples of one-half of Planck's constant:

$$1.0545887(57) \times 10^{-27}\ \text{erg} \cdot \text{s} = 6.582173(17) \times 10^{-22}\ \text{MeV}.$$

*Here and below we present the latest data. The numbers in brackets represent the uncertainty, equal to one standard deviation, in terms of the last digit of the main number, i.e. 0.5110034(14) = 0.5110034 ± 0.0000014.

Therefore, in the rational system of units, which we use, the spins of the electron and the proton turn out to be equal to one-half, and the spin of the photon (γ-quantum) is one:

$$s_e = s_p = {}^1/_2, \qquad s_\gamma = 1.$$

A third cardinal property of particles is the existence of electrical charge, the values of which are also quantized. The charges of all observed† particles are multiples of the so-called "elementary charge" $e = 4.803242(14) \times 10^{-10}$ CGS units $= 1.6021892(46) \times 10^{-19}$ C, equal to the charge of the electron. Unlike the spin, the quantum nature of which is quite clear, the discreteness of electrical charge represents an exciting mystery.

Finally, an important characteristic of a particle is its lifetime τ. The point is that only a few particles in the free state live for an indefinitely long time (i.e., are absoutely stable). Among these are the electron e^-, the proton p, the photon, and the neutrino ν,* and also their antiparticles—the positron e^+, the antiproton $\bar{p}$, and the antineutrino $\bar{\nu}$. All other particles are unstable and spontaneously decay, by the exponential law $\exp(-t/\tau)$, into other particles. The coefficient τ is called the lifetime. For example, the lifetime of the neutron, which decays according to the scheme (β-decay)

$$n \to p + e^- + \bar{\nu}_e,$$

turns out to be equal to $\tau_n = 918(14)$ s $= 15.3$ min. The charged pion π^+ decays into a muon and a neutrino:

$$\pi^+ \to \mu^+ + \nu_\mu$$

with a lifetime $\tau_{\pi^+} = 2.6030(23) \times 10^{-8}$ s, etc.

The instability of particles displays a most important property of the microcosm—the mutual transformation of particles into each other, which is a consequence of their interactions. The interactions of particles fall into four classes: strong, electromagnetic, weak, and gravitational. We shall not dwell in detail on the properties of the interactions (which are discussed below, in

†The electrical charges of quarks (hypothetical constituents of hadrons) are usually considered equal to simple fractions ($\pm\frac{1}{3}, \pm\frac{2}{3}$) of e. However, quarks in the free state have not been observed.

*Besides the two well-known types of neutrinos, the electronic ν_e and numonic ν_μ, there exists, probably, a third one—the ν_τ, a "partner" of the recently discovered heavy τ-lepton.

Section 10). We only point out that due to particle interactions, particles in some combination transform, as a rule, into other particles in some other combination, if such transitions are not forbidden by any conservation laws (of energy, momentum, angular momentum, electric charge, baryon number, strangeness, and some others).

The absence of any transition allowed by known conservation laws is considered to be an indication of the existence of some new, hitherto unknown conservation law.

1.2. Conservation laws. Generally speaking, conservation laws are the consequence of some symmetries, which reflect the property of nonobservability of certain characteristics of physical objects. It is well known that in conservative systems the law of energy conservation is the manifestation of symmetry with respect to the continuous operation of time translation. Invariance under time translation is, in turn, equivalent to the nonobservability of absolute time. Another sequence of the same type is made up by the parity conservation law, invariance under space inversion, and the conventionality of the concept of "right" and "left" (i.e., the nonobservability of "absolute right" and "absolute left").

Most of the conservation laws are connected with continuous symmetries and can be obtained from the latter with the aid of Noether's theorem (see below, Section 2.3). Among them the conservation laws of energy, momentum, and angular momentum may be singled out as following from symmetry of physical objects in the space-time. Such symmetry is a consequence of such deep and general properties as the nonobservability of absolute time and of absolute space coordinates (symmetry under space-time translations), the isotropy of space, and the equivalence of coordinate systems moving with respect to each other with constant velocities (symmetry under rotations in space and Lorentz rotations). The corresponding conservation laws are of a very general nature; they are characteristic of all particles and all interactions, and are obeyed in all transitions.

Within the accuracy of modern experiments the conservation laws of electric charge and baryon number are universal as well. These conservation laws can also be correlated with the already mentioned symmetries under phase transformations of the continuous type. The latter, however, have no explicit physical basis and are not connected with the space-time structure. Such symmetries are called *intrinsic symmetries.**

*Some recent theoretical speculations (models of grand unification of interactions) raise hope for the explanation of the quantum nature and conservation of the electric charge. At the same time, there seems to be a possibility of baryonic-charge nonconservation (in particular, in proton decay with lifetime $\geq 10^{31}$ years).

Table 1. *Interaction symmetry properties and conserved physical quantities*

Physical quantities	Interactions		
	Strong	Electromagnetic	Weak
Electric charge	+	+	+
Baryon number	+	+	+
Parity	+	+	−
Isospin	+	−	−
Strangeness	+	+	−

(+) = conserved; (−) = not conserved.

The class of intrinsic symmetries includes isotopic symmetry, and also some others (e.g., the so-called unitary symmetry). Most intrinsic symmetries and the related quantities conserved are of an approximate nature. With the two exceptions just mentioned (electric charge and baryon number) the respective conservation laws are obeyed in some interactions and violated in others. Isotopic invariance (the isotopic-spin conservation law) holds in strong interactions and is violated in electromagnetic and weak interactions. The law of parity (to be more precise, of space-parity) conservation, which is connected with the symmetry of the wave function under space inversion, holds in strong and electromagnetic interactions and is violated in weak interactions. Analogous behaviour is displayed by the conservation laws of strangeness (as well as of the new quantum numbers, charm and beauty). These properties are summarized in Table 1.

We shall not discuss herein the properties of the gravitational interaction, because of its very low intensity. For illustration we may point out that two protons are attracted with a gravitational force approximately 10^{37} times less than the force of their electric repulsion.

1.3. Particle–field correspondence. A field represents a physical system with an infinite number of degrees of freedom. The concept of field arises naturally when one attempts to reject the notion of instantaneous action of particles on each other at a distance (Newton's *actio in distans*)—a notion that contradicts the theory of special relativity (see Medvediev 1977, Part II, Section 6, and also Wigner 1971, p. 12). Considering the space between particles to be filled with a field, we charge the field with the task of transferring perturbations with finite speed from one particle to another. Thus, the introduction of classical fields into physics is dictated by reasons of relativistic invariance. In particle theory a central role is played by relativistic quantum fields.

The quantized wave field is a fundamental physical concept within the framework of which the dynamics of particles and their interactions is formulated. It allows one to describe various states of a system of many particles by means of a single physical object in ordinary space-time. Quantized fields appear when classical fields are quantized, as a result of which the field functions take on an operator meaning and are expressed in terms of particle creation and annihilation operators. Thereby there arises the possibility of describing a most important property of elementary particles—processes of mutual transformations.

An example, well known for a long time, of a classical field is the electromagnetic field, which describes light and at the same time the interaction of electrically charged particles. A classical description, based on Maxwell's equations, leads to a purely wave concept of electromagnetism. From a methodological viewpoint it sometimes turns out to be convenient to consider a continuous system—a field—as the limit of a discrete mechanical system with a number of degrees of freedom N, tending to infinity. One field oscillator corresponds to each degree of freedom.

A description of the corpuscular properties fo light is achieved as a result of a quantization procedure, in the course of which the field is associated with discrete energy quanta, corresponding to various possible energy states of the field oscillators. Quanta of the electromagnetic field—photons—have zero rest mass, have no electrical charge, and have a spin equal to one. The last fact corresponds to the polarization properties of classical fields and manifests itself in the fact that the electromagnetic field is multicomponent and is described by a set of field functions—the components of the electric or magnetic field strengths, or the components of the potential A_ν.

The properties of field functions, corresponding to other particles, also reflect the spin, charge, and other discrete characteristics of the respective particles. After quantization the quanta of the fields are usually identified with the particles.

To the real universe of interacting particles a system of coupled equations for the various fields is ascribed. After quantization the expressions representing the coupling of these equations (the interaction Lagrangian or Hamiltonian—see Section 10 below) describe elementary interactions between particles. These expressions acquire a clear interpretation in Feynman's rules of correspondence (see Sections 19, 20 below).

We shall postpone consideration of the interaction apparatus of particles and fields, and begin by examining properties of free fields and their quantization.

1.4. The representation of the Lorentz group. Let us consider the laws of transformation of field functions under relativistic transformations of coordinates. We recall some definitions.

The full (in other words, general orthochronous) *Lorentz group* consists of homogeneous linear transformations of the four coordiantes x^ν which leave invariant the quadratic form

$$x^2 = x_\nu x^\nu = (x^0)^2 - \mathbf{x}^2$$

and do not reverse the direction of time x^0. This group includes the usual spatial rotations, the Lorentz "rotations" in the x^0x^1, x^0x^2, and x^0x^3 planes (i.e., transformations under transitions to a coordinate system moving with respect to the initial one with constant velocity), the reflections of the three space axes, and all the products of these transformations. If time reversal is added, then we obtain the general Lorentz group.

In physics an important role is played by the group which consists of the full-Lorentz-group transformations together with translations along all four coordinate axes. This group is called the inhomogeneous Lorentz group (or the full *Poincaré group*). We shall call invariance under this group relativistic invariance, to be distinguished from Lorentz invariance, which corresponds to the Lorentz group.

Henceforth we shall be interested in the transformtion laws of field functions under transformations of coordinates belonging to the full Poincaré group

$$x \to x' = P\,(\omega;\; x), \tag{1}$$

where $\omega = (L,\, a)$ denotes the set of parameters describing translations (a) and rotations (L), and

$$x'_\nu = L_{\nu\mu}x^\mu + a_\nu, \qquad L_{\nu\sigma}g^{\sigma\rho}L_{\mu\rho} = g_{\nu\mu}. \tag{2}$$

The field function $u(x)$ represents either one function (being a single-component function) or several functions (being a many-component function) of the four coordinates x^ν defined in each of the reference frames. A transition from one coordinate system x to another x' which is related to x by the transformation (2) corresponds to a linear homogeneous transformation of the components of the field functions

$$u\,(x) \to u'\,(x') = \Lambda\,(\omega)\;u\,(x), \tag{3}$$

where the transformation matrix Λ is completely determined by the matrix L of the Lorentz transformation (2), i.e., depends on the same parameters as L.

We emphasize that the transformation (3) can not be reduced to the replacement of the argument x by x', since it describes a transformation from one coordinate system to another, and not a displacement from one point of space to another.

To each Lorentz transformation L there corresponds a matrix Λ_L, to the unit element of the group L there corresponds the unit matrix $\Lambda = 1$, while to the product of two elements L_1 and L_2 of the Lorentz group there corresponds the product

$$\Lambda_{L_1L_2} = \Lambda_{L_1}\Lambda_{L_2}.$$

A system of matrices with such properties is called, in group theory, a linear representation of the group. Matrices Λ of finite rank form a finite-dimensional representation of the Lorentz group. The rank of a representation is determined by the dimension of the matrices, i.e., by the number of components of u.

Therefore, the possible kinds of wave functions and the laws of their transformation can be obtained by investigation of the finite-dimensional (irreducible) representations of the Lorentz group. Such an investigation represents a special part of the theory of group representations, the result of which comes to the following. The finite-dimensional representations of the Lorentz group may be single-valued or double-valued, i.e., the correspondence $L \to \Lambda_L$ may be single-valued or double-valued. The importance to physics of double-valued representations is due to the fact that the field functions, generally speaking, are not directly observable (specifically, fields which transform according to double-valued representations always enter into the observable quantities in the form of quadratic combinations). However, the lack of single-valuedness of the operator Λ_L must be such that the observables transform in a completely unique manner under any Lorentz transformation L. Besides, it is necessary that the operators Λ_L be continuous functions of the parameters of the transformation L, i.e., that to an infinitesimal transformation of the reference frame there correspond an infinitesimal transformation of the field functions. The combined requirements stated above lead to the representations of the Lorentz group being divided into two categories. The first category is characterized by the single-valuedness of the correspondence $L \to \Lambda_L$ and contains the single-valued so-called *tensor* and *pseudotensor** representations. The field functions which

*The difference between tensors and pseudotensors is related to the reflections of space axes and will be considered at the end of this section.

transform in accordance with the tensor representation are called tensors (Pseudotensors) and in some cases may be directly observable (e.g. the electromagnetic field). The second category is characterized by this correspondence being double-valued: $L \to \pm\Lambda_L$.

The transformation law for a (pseudo) tensor of the Nth rank, $T^{\nu_1,\nu_2,\dots,\nu_N}$, under continuous transformations of coordinates has the form

$$T'_{\nu_1,\dots\nu_N}(x') = \frac{\partial x'_{\nu_1}}{\partial x_{\mu_1}}\cdots\frac{\partial x'_{\nu_N}}{\partial x_{\mu_N}} T_{\mu_1,\dots\mu_N}(x), \qquad (4)$$

or, using the notation (2),

$$T'^{\nu_1,\dots,\nu_N}(x') = L^{\nu_1}_{\mu_1}\dots L^{\nu_N}_{\mu_N} T^{\mu_1,\dots,\mu_N}(x).$$

The double-valued representations are called spinor representations, and the corresponding quantities are called *spinors*. The transformation law for spinor quantities has a more complex structure and is given for the simplest spinors in Appendix II. Here, we merely note that under the translation

$$x'^{\nu} = x^{\nu} + a^{\nu}$$

the transformation law for tensor quantities which follows from (4),

$$u(x) \to u'(x') = u(x) \qquad (5)$$

remains valid also for spinors.

We shall now present the simplest tensor representations and the quantities which correspond to them. The tensor of zero rank which under any continuous transformation transforms in accordance with the law (5) is an invariant and is called a scalar (pseudoscalar).

The tensor of the first rank which transforms under rotations of coordinates in accordance with the law

$$u'^{\nu}(x') = L^{\nu}_{\mu}u^{\mu}(x) = L^{\nu\mu}u_{\mu}(x), \qquad (6)$$

is called a contravariant vector (pseudovector). The covariant vector

$$u_{\nu}(x) = g_{\nu\mu}u^{\mu}(x)$$

connected with it transforms in accordance with the law

$$u'_\nu(x') = L^\mu_\nu u_\mu(x). \quad (7)$$

Corresponding formulas for covariant and contravariant tensors of the second and higher ranks may be written down without any difficulty.

Now consider the operation of space inversion P, i.e., of reflection of all three space axes:

$$x \to x' = Px, \quad x'_0 = x_0, \quad \boldsymbol{x}' = -\boldsymbol{x}. \quad (8)$$

The transformation laws of the field functions are not defined by the formulas (4) and must be formulated separately. Since the double application of the transformation ($P^2 = 1$) is equivalent to the identity operator due to the single-valuedness of the tensor representations, these laws for the components of the tensors $T^{\cdots}(x)$ can have only two forms:

$$PT(x) = T'(x') = T'(Px) = \pm T(x).$$

A tensor of zero rank which does not change sign under inversion,

$$Pu(x) = +u(x) \quad (9)$$

is called a scalar. In the other case, the corresponding quantity which satisfies the relation

$$Pu(x) = -u(x), \quad (10)$$

is called a pseudoscalar.

If a tensor of the first rank changes the signs of its space components under the P-transformation and does not change the sign of the time component:

$$PV^\mu(x) = V_\mu(x),$$

i.e.,

$$PV^0(x) = V^0(x), \quad P\boldsymbol{V}(x) = -\boldsymbol{V}(x), \quad (11)$$

it is then called a vector. If, on the other hand, only the time component changes sign and the space components do not:

$$PV^\mu(x) = -V_\mu(x),$$

i.e.,

$$PV^0(x) = -V^0(x), \quad P\boldsymbol{V}(x) = \boldsymbol{V}(x),$$

then it is a pseudovector (axial vector). Generally, it is possible to describe the transformation law of pseudotensors by the formula (4) multiplied by the determinant of the transformation of the coordinates. The relations (9), (10) and other similar relations define also the important property of *parity* of the field functions and of the respective particles. This characteristic plays an essential role in determining (see Section 10.2 below) the possible forms of interaction between various fields.