SOLID STATE PHYSICS

Problems and Solutions

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A WILEY-INTERSCIENCE PUBLICATION
JOHN WILEY & SONS, INC.
New York • Chichester • Brisbane • Toronto • Singapore

This text is printed on acid-free paper.

Library of Congress Cataloging in Publication Data:

Mihály, László, 1949–
Solid state physics : problems and solutions / László Mihály, Michael C. Martin
p. cm.
Includes bibliographical references and index.
ISBN 0-471-15287-0 (paper : alk. paper)
1. Sold state physics—Problems, exercises, etc. I. Martin, Michael C. II. Title.
QC176.515.M54 1996
530.4'1'076—dc20 96-6184

Printed in the United States of America

10 9 8 7 6 5 4 3 2

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Preface

A number of years ago, one of us had the opportunity to attend lectures by a particularly talented and dedicated high-school science teacher, Miklós Vermes. The lectures were full of interesting and entertaining demonstrations, and he told the audience a lot about how the basic laws of physics fit together into a logical structure. Yet, when Professor Vermes was asked about what is the really important part in learning physics, he answered: "Solve homework problems. That way, you will get used to the stuff. In fact, 'getting used to it' is a major part of learning."

In this spirit, we hand this book to advanced undergraduate and introductory graduate students in Condensed Matter Physics. "Get used to it." When you are used to it, when you no longer stumble over every detail, you will be able to see the forest for the trees more clearly.

We wrote this book to satisfy another need as well: to help measure the progress of the students at midterms, finals, and comprehensive exams. Every professor must have a set of carefully guarded problems, appropriate to the final exam of the course she/he is teaching (better yet to have several sets, if you are teaching in several consecutive semesters). One would like to share and discuss these—presumably interesting—physics questions with the students, but practical considerations do not allow for this; if a problem is given out as a homework assignment, its value as a final exam problem is greatly and understandably reduced. Our goal here was to provide a volume of problems greater in number than the "critical mass" number which can be memorized. If a student remembers *all* of the solutions for a sufficiently large set of problems, he or she pretty much knows the subject (for proof, see the argument in the first paragraph).

The problems and solutions presented in this book stem from several years of teaching advanced undergraduate and introductory graduate solid state physics courses in the Physics Department of SUNY at Stony Brook. During these courses we used several of the excellent textbooks available; some of them are listed as references to the present collection of problems. Naturally, the problems were developed and organized each year, more or less in accord with the textbook used in that particular course. As we began to assemble the present collection, we planned to divide the problems into chapters corresponding to one of the standard organizations of the subject matter. However, we discovered that we do not need to be tied by the same constraints as the typical introductory textbook: A particular aspect of superconductivity may be effectively used to illustrate the concept of density of states; magnetism and charge density waves fit reasonably well under the umbrella of interacting electron systems.

We recommend the use of this collection in conjunction with one of the "standard" textbooks. The textbook will provide the backbone of organization very much needed for the first-time encounter with the subject. The instructor should pick the appropriate problems and assign them (in tandem with the problems from the textbook) as the course proceeds. Presumably, by the end of the course the problems *not* assigned can be used by the students for review and integrating their knowledge. We also hope that the problems solved here will provide inspiration for creating other, similar problems.

Although both authors' primary interests are experimental in nature, the reader will find a few problems concerning model systems with little practical relevance. In addition, there were several times when we felt that a particular concept is better illustrated if the mathematics remains simple, so we utilized one- or two-dimensional systems (which are much easier to make graphical illustrations of too!). We hope that we do not draw too much criticism for not being realistic or for neglecting important practical experimental issues. The more abstract problems in this collection may help the community of condensed matter physicists to preserve a common language so that an experimenter can still attend a theory talk without losing track within the first two minutes. Nevertheless, we tried to keep this book at an introductory level, and the problems using second quantization formalism do not really require more than the knowledge of the basic commutation relations.

Many people helped us eliminate some (but, we are afraid, not all) conceptual and practical mistakes in this book. We are particularly indebted to Jenő Sólyom, Manuel Cardona, György Kriza, Attila Virosztek, and Gábor Oszlányi for helpful comments. We would also like to thank Vladimir Golovanov for a critical reading of our manuscript. We would appreciate comments from readers (you can find us on the Internet or contact us via the publisher).

Most of the graphs in this book were produced using CoPlot.[1] The typesetting was done with LaTeX and using the emTeX program.[2]

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May 1996

[1] "CoPlot," computer plotting software, © 1988 and 1990 by CoHort Software, Berkeley, California.

[2] emTeX, TeX processing software, written by Eberhard Mattes and available free of charge on the Internet.